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JHEP08(2009)002

(http://iopscience.iop.org/1126-6708/2009/08/002)

Published by IOP Publishing for SISSA

Received: *April 20, 2009*
Accepted: *July 16, 2009*
Published: *August 4, 2009*

# Probing Yukawa unification with K and B mixing

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Abstract: We consider corrections to the unification of down-quark and charged-lepton Yukawa couplings in supersymmetric GUTs, which links the large $\nu_\tau - \nu_\mu$ mixing angle to $b \to s$ transitions. These corrections generically occur in simple grand-unified models with small Higgs representations and affect $s \to d$ and $b \to d$ transitions via the mixing of the corresponding right-handed superpartners. On the basis of a specific SUSY-SO(10) model, we analyze the constraints from $K-\overline{K}$ and $B_d-\overline{B}_d$ mixing on the additional $\tilde{d}_R-\tilde{s}_R$ rotation angle $\theta$. We find that $\epsilon_K$ already sets a stringent bound on $\theta$, $\theta^{\max} \sim \mathcal{O}(1^\circ)$, indicating a very specific flavor structure of the correction operators. The impact of the large neutrino mixings on the unitarity triangle analysis is also briefly discussed, as well as their ability to account for the sizeable CP-violating phase observed recently in $B_s \to J/\psi\phi$ decays.

Keywords: B-Physics, Supersymmetric Standard Model, GUT, Kaon Physics

ArXiv ePrint: 0904.0378



doi:10.1088/1126-6708/2009/08/002

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## Contents



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## 1 Introduction

The start of the LHC at CERN will enable us to study TeV-scale physics directly for the first time. Most importantly, we will eventually probe the mechanism of electroweak symmetry breaking; moreover, we will be able to test various scenarios for new physics beyond the standard model (SM), the leading candidate of which is arguably supersymmetry (SUSY). The presence of supersymmetry at the TeV-scale eliminates the quadratically-divergent loop contributions to the Higgs mass and thereby stabilizes the electroweak scale against the scales of more fundamental physics. In addition, TeV-scale SUSY models provide an attractive mechanism for electroweak symmetry breaking and an appealing candidate for cold dark matter. Furthermore, they offer a compelling outline for the unification of all matter and interactions, the first step of which is grand unification [1].

The near unification of the SM gauge couplings within the minimal supersymmetric standard model (MSSM) at $M_{\rm GUT} \simeq 2 \cdot 10^{16}\,\text{GeV}$, with the MSSM being valid above the TeV-scale, suggests that the standard-model group, $\text{G}_{\rm SM} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$, is embedded into a simple gauge group at this scale, such as SU(5) [2] or SO(10) [3]. SO(10) is arguably the most natural GUT group: both the SM gauge and matter fields are

unified, introducing only one additional matter particle, the right-handed neutrino.[1] It is an anomaly-free theory and therefore explains the intricate cancellation of the anomalies in the standard model [4]. Moreover, it contains $B-L$ as a local symmetry, where $B$ and $L$ are baryon and lepton number, respectively; the breaking of $B-L$ provides light neutrino masses via the seesaw mechanism. Remarkably, $M_{\rm GUT}$ is of the right order of magnitude to generate neutrino masses in the sub-eV range. Hence, the neutrino masses are linked to the breaking of the GUT symmetry [5].

Flavor experiments, though not able to access the TeV scale directly, have put strong constraints on the MSSM parameters. Due to the lack of deviation with respect to the SM, we expect the new sources of flavor mixing and CP violation to be very limited for SUSY particles around the weak scale. As formulated by the concept of minimal flavor violation [6, 7], we assume that the Yukawa couplings are the only source of flavor violation and (even more) that the supersymmetry breaking parameters are universal at some fundamental scale. Within the minimal supergravity (mSUGRA) scenario [8], this scale is usually taken to be $M_{\rm GUT}$. An alternative and arguably more natural choice, however, would be the Planck scale, $M_{\rm Pl} = G_N^{-1/2} = 1 \cdot 10^{19}\,\text{GeV}$.[2] The reason to take the MSSM unification scale instead is simply that while the use of the renormalization group equations of the MSSM below $M_{\rm GUT}$ is undisputed, the analysis of the region between $M_{\rm GUT}$ and $M_{\rm Pl}$ requires knowledge about the grand-unified model. However, the universality of the SUSY-breaking parameters is broken by their evolution down to lower energies. Thus the choice of $M_{\rm GUT}$ eliminates potentially important flavor effects. In our analysis, we will adopt $M_{\rm Pl}$ as universality scale, and study consequences of this choice in detail.

In the standard model, fermion mixing is only measurable among the left-handed states and described by the quark and lepton mixing matrices, $V_{\rm CKM}$ and $V_{\rm PMNS}$. Both small and large mixing angles are realized: while those in the quark sector are small, two angles in $V_{\rm PMNS}$ turn out to be large. These are the neutrino solar and atmospheric mixing angles, where the latter is close to maximal. The effects of $V_{\rm CKM}$ and $V_{\rm PMNS}$ are confined to the quark and to the lepton sectors, respectively. In GUTs, however, this separation of quark and lepton sector is abrogated as quarks and leptons are unified. Thus their masses and mixings are related to each other. While different patterns are possible, it is natural to expect imprints of $V_{\rm PMNS}$ on the quark sector as well. In particular, it might be possible to trade off small rotations of left-handed down quarks and right-handed leptons against large mixings among right-handed down quarks and left-handed leptons, as we will discuss below. The mixing of the right-handed fermions is unobservable due to the absence of right-handed flavor-changing currents at the weak scale. With weak-scale supersymmetry, the mixing of the corresponding scalar partners of quarks and leptons becomes physical.

The impact of the large atmospheric mixing angle on $B_s$ physics has already been investigated in detail [9–12]. Due to the good agreement of the bottom-quark and tau-lepton masses at $M_{\rm GUT}$, one can adopt the predicted Yukawa unification of down quarks and charged leptons. In order to study $K$ and $B_d$ physics, however, one needs to go beyond

[1] Strictly speaking, it is the left-handed neutrino singlet.

[2] Alternatively, one might choose the *reduced* Planck scale, $M_{\rm Pl} = (8\pi G_N)^{-1/2} = 2 \cdot 10^{18}\,\text{GeV}$, because it compensates for the factor $8\pi$ in the Einstein field equations.



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minimal models and modify the relations among the Yukawa couplings.[3] Here, we can pursue two avenues: we can either introduce additional Higgs fields in larger representations, such as a $45_H$ in SU(5), or parameterize the modifications via higher-dimensional operators, suppressed by powers of a more fundamental scale [15, 16]. We opt for the latter route for three reasons. One, large Higgs representations introduce a large number of additional fields, which both yields large threshold corrections at $M_{\rm GUT}$ and makes the gauge coupling blow up shortly above the GUT scale. Two, the use of higher-dimensional operators reflects the successful Yukawa unification of the third generation; the corrections are suppressed and therefore apply mostly to the lighter generations. Finally, we are able to perform a more general study as we do not rely on specific Higgs fields.

In this paper, we will study the impact of the higher-dimensional Yukawa operators on $K - \overline{K}$ and $B_d - \overline{B}_d$ mixing. A SUSY-SO(10) GUT with universal supersymmetry-breaking parameters at the Planck scale will serve as our specific framework. In particular, the precise measurement of $\epsilon_K$ will enable us to tightly constrain the additional (s)quark mixing caused by these operators. The validity of our results for more general classes of grand-unified models will also be assessed.

## 2 Yukawa unification and dimension-five operators

Grand-unified theories using small Higgs representations to break the electroweak symmetry generically predict the unification of down-quark and charged-lepton masses [1, 2].[4] Before turning to SO(10), let us consider minimal SU(5) to bring out the central idea of this work. Here the down-quark singlet, $d^c$, and lepton doublet, $L$, fill up the $\overline{5}$ representation, whereas the quark doublet, $Q$, as well as the up-quark and the electron singlets, $u^c$ and $e^c$, are embedded in the 10. As usual, these are left-chiral superfields; for instance, we have the electron singlet $e^c_L$ instead of the right-handed electron $e_R$. The adjoint Higgs field $\Sigma$ breaks SU(5) to the standard-model group, which is then broken to $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\rm em}$ by a pair of quintets, $H + \overline{H}$.

The corresponding Yukawa couplings read

$$W_Y = \mathsf{Y}_1^{ij}\, \epsilon_{abcde}\, 10_i^{ab}\, 10_j^{cd}\, H^e + \mathsf{Y}_2^{ij}\, 10_i^{ab}\, \overline{5}_{ja}\, \overline{H}_b \,, \tag{2.1}$$

where $a, b, \ldots$ denote SU(5) and $i, j$ flavor indices. The second coupling yields the unification of down-quark and charged-lepton Yukawa couplings $\mathsf{Y}_{d,e}$ (and thus of the corresponding masses). If $\mathsf{Y}_{d,e}$ are defined such that the weak doublets are on the left and the singlets on the right, we obtain

$$\mathsf{Y}_d = \mathsf{Y}_e^\top = \mathsf{Y}_2 \,. \tag{2.2}$$

[3] These modifications were neglected in ref. [13], whose authors consider minimal SU(5). Similarly, ref. [14] assumes a minimal SO(10) model where $V_{\rm CKM}$ describes all SM flavor mixing (the study is from 1995, i.e. before the large mixing angles in the lepton sector were established).

[4] The unification of down-quark and charged-lepton masses is a prediction of the SU(4) symmetry, which is present in the Pati-Salam model and respected in minimal SU(5).



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The mixings of the right-handed (left-handed) down quarks are thus identical (or, more precisely, conjugated) to those of the left-handed (right-handed) charged leptons.

This relation works remarkably well for the third generation but not for the lighter ones. Thus we need to include corrections, which are generically generated by higher-dimensional Yukawa operators, suppressed by powers of the Planck scale, $M_{\rm Pl}$ [15]. With the given particle content, we have two operators of mass-dimension five contributing to the down-quark and charged-lepton masses [15],

$$\mathsf{Y}^{ij}_{\sigma 1}\, 10^{ab}_i\, \overline{5}_{ja}\, \frac{\Sigma^c_b}{M_{\rm Pl}}\, \overline{H}_c + \mathsf{Y}^{ij}_{\sigma 2}\, 10^{ab}_i\, \overline{5}_{jc}\, \frac{\Sigma^c_b}{M_{\rm Pl}}\, \overline{H}_a \ . \tag{2.3}$$

The vacuum expectation value (vev) of $\Sigma$ is proportional to hypercharge, $\langle\Sigma\rangle = \sigma\, \mathrm{diag}\,(2,2,2;-3,-3)$. Hence, the second operator modifies the relation (2.2),

$$\mathsf{Y}_d = \mathsf{Y}_e^\top + 5\frac{\sigma}{M_{\rm Pl}}\, \mathsf{Y}_{\sigma 2} \ . \tag{2.4}$$

Now we cannot diagonalize both Yukawa matrices simultaneously anymore. In the basis where the charged leptons are diagonal, we obtain

$$L_d\, \mathsf{D}_d R_d^\dagger = \mathsf{D}_e + 5\frac{\sigma}{M_{\rm Pl}}\, \widetilde{\mathsf{Y}}_{\sigma 2} \ ; \tag{2.5}$$

$\mathsf{D}_i$ denote the diagonal Yukawa matrices, and $L_d$ and $R_d$ are unitary rotation matrices for the down-quark fields. The good agreement of the bottom and tau masses at the GUT scale indicates that the rotation matrices $L_d$ and $R_d$ have a non-trivial 1-2 block only,[5]

$$L_d,\, R_d \sim \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{2.6}$$

Hence, the effect of the additional rotations may only be seen in observables involving the first and second generations.

The effect of the dimension-five operators on proton decay has been studied in great detail [17]. In this paper, we point out that the rotation matrix $R_d$ can be severely constrained by the precise measurements in $K$ and $B_d$ physics. This, in turn, allows for a complementary study of these operators and thus enables us to probe grand-unified models.

In the following, we will omit the indices of the higher-dimensional operators. For instance, we will denote the operators in eq. (2.3)

$$\mathsf{Y}^{ij}_{\sigma}\, 10_i\, \overline{5}_j\, \frac{\Sigma}{M_{\rm Pl}}\, \overline{H} \equiv \mathsf{Y}^{ij}_{\sigma 1}\, 10^{ab}_i\, \overline{5}_{ja}\, \frac{\Sigma^c_b}{M_{\rm Pl}}\, \overline{H}_c + \mathsf{Y}^{ij}_{\sigma 2}\, 10^{ab}_i\, \overline{5}_{jc}\, \frac{\Sigma^c_b}{M_{\rm Pl}}\, \overline{H}_a \ . \tag{2.7}$$

Note that these index-less operators represent all possible combinations for the fields to form a singlet, and so $\mathsf{Y}_\sigma$ is an *effective* coupling matrix.

[5] Even if $\mathsf{Y}^{33}_{\sigma 2} \sim 1$, it is suppressed with respect to $\mathsf{Y}^{33}_2$ by $\sigma/M_{\rm Pl}$.



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## 3 Framework

Let us now turn to SO(10) and consider a model proposed by Chang, Masiero, and Murayama (CMM) [9]. Here the matter fields are unified in the spinor representations, $16_i$, together with the right-handed neutrinos. SO(10) is broken to SU(5) by a pair of Higgs spinors, $16_H + \overline{16}_H$. Next, an adjoint field, $45_H$, breaks SU(5) and the electroweak symmetry is eventually broken by a pair of fundamental Higgs fields, $10_H$ and $10'_H$. In fact, both the SU(5) adjoint and the SU(5) singlet of $45_H$ acquire vevs, the latter (denoted by $v_0 \sim 10^{17}\,\text{GeV}$) being an order of magnitude larger than the former ($\sigma \sim 10^{16}\,\text{GeV}$).

The Yukawa couplings in the CMM superpotential read

$$W_Y = 16_i\,\mathsf{Y}_1^{ij}\,16_j\,10_H + 16_i\,\mathsf{Y}_2^{ij}\,16_j\,\frac{45_H\,10'_H}{M_{\text{Pl}}} + 16_i\,\mathsf{Y}_N^{ij}\,16_j\frac{\overline{16}_H\overline{16}_H}{M_{\text{Pl}}}\ . \tag{3.1}$$

Let us discuss the individual terms in detail. In the fundamental Higgs field $10_H$, only the up-type Higgs doublet $H_u$ acquires a weak-scale vev such that the first term gives masses to the up quarks and neutrinos only. The masses for the down quarks and charged leptons are then generated through the vev of the down-type Higgs doublet $H_d$ in the second fundamental Higgs field $10'_H$. (A second Higgs field with Yukawa couplings to the SM fermions is generally needed in order to have a non-trivial flavor structure.) They are obtained from the second term in eq. (3.1) which is of mass-dimension five, in contrast to minimal SU(5). As indicated above, this operator actually stands for various, inequivalent effective operators with both the SU(5)-singlet and the SU(5)-adjoint vevs of the adjoint Higgs field $45_H$ such that the coupling matrix $\mathsf{Y}_2$ can only be understood symbolically. The magnitude of this second mass term is determined by the vev of the SU(5)-singlet component, $v_0$, which contributes equally to down-quark and charged-lepton masses. The strong hierarchy between the $t$ and $b, \tau$ masses then follows from the $v_0/M_{\text{Pl}}$ suppression factor. The smaller SU(5)-breaking vev ($\sigma$), which is proportional to hypercharge as in SU(5), will be important for the modification of the light generation Yukawa couplings. The second term in eq. (3.1) can be constructed in various ways, for example by integrating out SO(10) fields at the Planck scale. The corresponding couplings can be symmetric or antisymmetric [18], resulting in an asymmetric effective coupling matrix $\mathsf{Y}_2$, as opposed to the symmetric matrices $\mathsf{Y}_1$ and $\mathsf{Y}_N$. Finally, the third term in eq. (3.1), again a higher-dimensional operator, generates Majorana masses for the right-handed neutrinos.

We can always choose a basis where one of the Yukawa matrices in eq. (3.1) is diagonal. In particular, the basis where $\mathsf{Y}_1^{ij}$ is diagonal will be referred to as the up-basis. In the CMM model, however, one assumes that $\mathsf{Y}_1^{ij}$ and $\mathsf{Y}_N^{ij}$ are simultaneously diagonalizable. This assumption is motivated by the observed values for the fermion masses and mixings and might be a result of family symmetries. First, we note that the up quarks have a stronger hierarchy than the down quarks, charged leptons, and neutrinos. Consequently, the eigenvalues of $\mathsf{Y}_N$ must almost have a double hierarchy compared to $\mathsf{Y}_1$. Then, given the Yukawa couplings in an arbitrary basis, we expect smaller off-diagonal entries in the rotation matrices of $\mathsf{Y}_1$ and $\mathsf{Y}_N$ than in $\mathsf{Y}_2$ because hierarchical masses generically correspond to small mixing. Moreover, the light neutrino mass matrix implies that, barring cancellations, the rotations needed to diagonalize $\mathsf{Y}_1$ should be smaller than those in $V_{\text{CKM}}$ [19].



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Thus, even if $\mathsf{Y}_N$ is not exactly diagonal in the up-basis, the off-diagonal entries in its rotation matrix will be much smaller than the entries in $V_{\rm CKM}$ so that they cannot spoil the large mixings among $d_R$ quarks generated by $V_{\rm PMNS}$.

Now, with $\mathsf{Y}_1$ and $\mathsf{Y}_N$ being simultaneously diagonal, the flavor structure is (apart from supersymmetry-breaking terms, which we will discuss below) fully contained in the remaining coupling, $\mathsf{Y}_2$. Let us assume for the moment that the relation (2.2) is valid. Then we can rewrite the superpotential in the SU(5) basis as

$$W_Y = 16_i\,\mathsf{D}_1^{ij}\,16_j\,10_H + 16_i\left(V_q^*\mathsf{D}_2 V_\ell\right)^{ij}\,16_j\,\frac{45_H\,10'_H}{M_{\rm Pl}} + 16_i\,\mathsf{D}_N^{ij}\,16_j\frac{\overline{16}_H\overline{16}_H}{M_{\rm Pl}}\,, \tag{3.2}$$

where the second coupling is to be understood as $(Q,e^c)^\top\,V_q^*\mathsf{D}_2V_\ell\,(d^c,L)\,45_H10'_H/M_{\rm Pl}$ (cf. section 2). Then $V_q$ and $V_\ell$ coincide with the quark and lepton mixing matrices, $V_{\rm CKM}$ and $V_{\rm PMNS}$, up to phases.[6] Note that the mass matrices of both down quarks and charged leptons have a lopsided structure.

As discussed in the previous section, the relation (2.2) needs to be modified. Using the SU(5)-breaking vev of $45_H$, $\sigma$, we obtain

$$\mathsf{Y}_d = \mathsf{Y}_e^\top + 5\frac{\sigma}{v_0}\,\mathsf{Y}_\sigma\,, \tag{3.3}$$

in accordance with SU(5) discussed above. Again, this notation is symbolic, as $\mathsf{Y}_\sigma$ stems from several distinct operators. Without these corrections, the large atmospheric mixing angle could directly be translated to maximal mixing between the right-handed down squarks $\tilde b_R$ and $\tilde s_R$. Now the CKM matrix diagonalizes $\mathsf{Y}_d\,\mathsf{Y}_d^\dagger$ whereas the PMNS matrix diagonalizes $\mathsf{Y}_e\,\mathsf{Y}_e^\dagger$, such that we cannot give a general relation between the contributions of the correction operators and additional rotations. Let us therefore make the ansatz

$$R_d = (U\,V_\ell)^\top\,, \tag{3.4}$$

i.e., the rotation of the down-quark singlet fields differs from that of the lepton doublets by a unitary matrix $U$. Clearly, in absence of the correction operators, $U=\mathbb{1}$. As said before, the goal of this paper is to study how much the rotations parameterized by $R_d$ differ from those of the charged leptons, i.e. whether a sizeable admixture of $\tilde d_R$ in $\tilde s'_R$ is allowed.

As discussed above, the good bottom-tau unification implies that the (33)-entry of $U$ should be close to one, up to a phase, and the remaining entries of the third row and column should be small. Thus we parameterize $U$ as

$$U = \begin{pmatrix} U_{11} & U_{12} & 0 \\ U_{21} & U_{22} & 0 \\ 0 & 0 & e^{i\phi_4} \end{pmatrix} = \begin{pmatrix} \cos\theta\,e^{i\phi_1} & -\sin\theta\,e^{i(\phi_1-\phi_2+\phi_3)} & 0 \\ \sin\theta\,e^{i\phi_2} & \cos\theta\,e^{i\phi_3} & 0 \\ 0 & 0 & e^{i\phi_4} \end{pmatrix}, \tag{3.5}$$

with $\theta\in[0,\pi/2]$. For concreteness, let us assume the tribimaximal form for the leptonic mixing matrix, corresponding to the mixing angles $\theta_{12}=\arcsin\left(1/\sqrt{3}\right)\simeq 35^\circ$, $\theta_{13}=0^\circ$,

[6] In the up-basis, $V_{\rm CKM}$ is conventionally defined as the matrix that rotates the left-handed down-quark mass eigenstates into the weak eigenbasis, while the inverse of $V_{\rm PMNS}$ rotates the corresponding charged leptons. The transposition between $R_d$ and $V_\ell$ in eq. (3.4) is due to relation (2.2).



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and $\theta_{23} = 45°$. In the up-basis, we can have $V_q$ in its standard parametrization and thereby absorb five of the six phases. Then we can indeed identify $V_q = V_{\rm CKM}$. We cannot do so for $V_\ell$ since we would only move the phases from the down-quark Yukawa matrix to the down-squark soft-breaking masses. We therefore choose to have $V_\ell$ with six phases; to see them explicitly, let us write down the mixing matrix for $\theta_{13} \neq 0$,

$$V_\ell = \begin{pmatrix} \sqrt{\frac{2}{3}} c_{13}\, e^{i\alpha_1} & \frac{1}{\sqrt{3}} c_{13}\, e^{i\alpha_2} & s_{13}\, e^{i(\delta+\alpha_3)} \\ e^{i\alpha_4}\left(-\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} s_{13}\, e^{-i\delta}\right) & e^{i(-\alpha_1+\alpha_2+\alpha_4)}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} s_{13}\, e^{-i\delta}\right) & \frac{1}{\sqrt{2}} c_{13}\, e^{i(-\alpha_1+\alpha_3+\alpha_4)} \\ e^{i\alpha_5}\left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} s_{13}\, e^{-i\delta}\right) & e^{i(-\alpha_1+\alpha_2+\alpha_5)}\left(-\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} s_{13}\, e^{-i\delta}\right) & \frac{1}{\sqrt{2}} c_{13}\, e^{i(-\alpha_1+\alpha_3+\alpha_5)} \end{pmatrix}, \tag{3.6}$$

where $c_{13} = \cos\theta_{13}$ and $s_{13} = \sin\theta_{13}$. In this parametrization, we can easily identify the standard phase, $\delta$, and then the standard form for $V_{\rm PMNS}$ is given by $V_{\rm PMNS} = P_L V_\ell P_R$, where

$$P_L = \mathrm{diag}\left(e^{-i\alpha_1},\, e^{-i\alpha_4},\, e^{-i\alpha_5}\right), \qquad P_R = \mathrm{diag}\left(1,\, e^{i(\alpha_1-\alpha_2)},\, e^{i(\alpha_1-\alpha_3)}\right). \tag{3.7}$$

If $\theta_{13}$ is indeed zero, the phase $\delta$ drops out of the matrix (3.6). This situation is familiar from the standard model: CP violation requires $\theta_{13} \neq 0$. Altogether, for $\theta_{13} = 0$, the mixing matrix for the right-handed down quarks in eq. (3.4) reads

$$R_d = \frac{1}{\sqrt{6}} \begin{pmatrix} 2U_{11}\, e^{i\alpha_1} - U_{12}\, e^{i\alpha_4} & 2U_{21}\, e^{i\alpha_1} - U_{22}\, e^{i\alpha_4} & e^{i(\phi_4+\alpha_5)} \\ \sqrt{2}\, e^{i\alpha_2}\left(U_{11} + U_{12}\, e^{i(\alpha_4-\alpha_1)}\right) & \sqrt{2}\, e^{i\alpha_2}\left(U_{21} + U_{22}\, e^{i(\alpha_4-\alpha_1)}\right) & -\sqrt{2}\, e^{i(\phi_4-\alpha_1+\alpha_2+\alpha_5)} \\ \sqrt{3} U_{12}\, e^{i(-\alpha_1+\alpha_3+\alpha_4)} & \sqrt{3} U_{22}\, e^{i(-\alpha_1+\alpha_3+\alpha_4)} & \sqrt{3}\, e^{i(\phi_4-\alpha_1+\alpha_3+\alpha_5)} \end{pmatrix} \tag{3.8}$$

with $U_{ij}$ as given in eq. (3.5).

Due to the absence of right-handed multiplets in the standard model, mixing among the right-handed down quarks is unobservable. With supersymmetry, however, the mixing of the corresponding squarks potentially leads to enhanced amplitudes for flavor-changing processes. As mentioned in the Introduction, we will assume universal soft-breaking terms at the Planck scale. This universality, however, is no longer present at the electroweak scale. For the scalar masses, this is due to the large Yukawa coupling of the third generation in the renormalization group evolution (RGE), such that

$$\mathsf{M}^2_{\tilde{d}}(M_Z) = \mathrm{diag}\left(m^2_{\tilde{d}},\; m^2_{\tilde{d}},\; m^2_{\tilde{d}}\left(1 - \Delta_{\tilde{d}}\right)\right) \tag{3.9}$$

in the case of the $\tilde{d}_R$ soft-breaking terms. The fast RGE between $M_{\rm Pl}$ and $v_0$ allows for rather large values of $\Delta_{\tilde{d}}$ [9, 11]. Now choosing the super-CKM basis where the down quarks are mass eigenstates, this matrix is no longer diagonal; in particular, all elements of the 2-3 block are of comparable size:

$$\widetilde{\mathsf{M}}^2_{\tilde{d}} = R_d^\dagger\, \mathsf{M}^2_{\tilde{d}}\, R_d = m^2_{\tilde{d}} \begin{pmatrix} 1 - \sin^2\theta\, \Delta_{\tilde{d}}/2 & \sin(2\theta)\, e^{-i\phi_K}\, \Delta_{\tilde{d}}/4 & \sin\theta\, e^{-i\phi_{B_d}}\, \Delta_{\tilde{d}}/2 \\ \sin(2\theta)\, e^{i\phi_K}\, \Delta_{\tilde{d}}/4 & 1 - \cos^2\theta\, \Delta_{\tilde{d}}/2 & -\cos\theta\, e^{-i\phi_{B_s}}\, \Delta_{\tilde{d}}/2 \\ \sin\theta\, e^{i\phi_{B_d}}\, \Delta_{\tilde{d}}/2 & -\cos\theta\, e^{i\phi_{B_s}}\, \Delta_{\tilde{d}}/2 & 1 - \Delta_{\tilde{d}}/2 \end{pmatrix},$$

$$\phi_K = \phi_1 - \phi_2\,, \qquad \phi_{B_s} = \phi_3 - \phi_4 + \alpha_4 - \alpha_5\,, \qquad \phi_{B_d} = \phi_1 - \phi_2 + \phi_3 - \phi_4 + \alpha_4 - \alpha_5\,. \tag{3.10}$$



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This observation motivated detailed studies of $b \to s$ transitions in supersymmetric GUT models, in particular the decay $b \to s\gamma$ and $B_s - \overline{B}_s$ mixing [9–11]. In the following, we will study the impact of the 1-2 and 1-3 blocks, generated by the angle $\theta$ in eq. (3.5), on the analogous $s \to d$ and $b \to d$ transitions, focussing on $K - \overline{K}$ and $B_d - \overline{B}_d$ mixing.

## 4 Meson-antimeson mixing

The oscillations of a $P^0 - \overline{P}^0$ meson system can be described by a Schrödinger-type equation,

$$i\frac{d}{dt}\begin{pmatrix}|P^0(t)\rangle\\ |\overline{P}^0(t)\rangle\end{pmatrix} = \left[M^P - \frac{i}{2}\Gamma^P\right]\begin{pmatrix}|P^0(t)\rangle\\ |\overline{P}^0(t)\rangle\end{pmatrix}, \tag{4.1}$$

where $M^P$ and $\Gamma^P$ are two $2 \times 2$ hermitian matrices which encode the four transitions $P^0/\overline{P}^0 \to P^0/\overline{P}^0$ via virtual and physical intermediate states, respectively. The physical states $|P_1^0\rangle$ and $|P_2^0\rangle$ are obtained by diagonalizing $M^P - \frac{i}{2}\Gamma^P$. The relevant quantity to study new-physics effects in $P^0 - \overline{P}^0$ mixing is the local contribution to the off-diagonal element of $M^P$:

$$M_{12}^P = \frac{1}{2M_P}\left\langle P^0\middle|\mathcal{H}_{\text{eff}}^{\Delta F=2}\middle|\overline{P}^0\right\rangle, \tag{4.2}$$

with $M_P$, the average meson mass $(M_{P_1} + M_{P_2})/2$. The effective Hamiltonian $\mathcal{H}_{\text{eff}}^{\Delta F=2}$, which comprises in general eight effective operators,

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \frac{G_F^2 M_W^2}{16\pi^2}\sum_{i=1}^{8} C_P^i(\mu_P)\, Q_P^i(\mu_P), \tag{4.3}$$

is conveniently expressed at the scale $\mu_P \sim M_P$ in the $B_d$ and $B_s$ systems, and at the scale $\mu_P \lesssim m_c$ in the kaon system. For an extensive introduction into the formalism of $K - \overline{K}$ and $B_{d,s} - \overline{B}_{d,s}$ mixing, see e.g. ref. [20].

One observable which is particularly well-suited to constrain the additional rotation of the $\tilde{d}_R$ and $\tilde{s}_R$ squarks in eq. (3.5) is

$$|\epsilon_K| = \kappa_\epsilon \frac{\text{Im}\left(M_{12}^K\right)}{\sqrt{2}\Delta M_K}, \tag{4.4}$$

which measures the amount of CP-violation in $K - \overline{K}$ mixing amplitudes. Indeed, $|\epsilon_K|$ is very small in the standard model and its experimental value, measured with high precision, leaves only little room for new physics. The correction factor $\kappa_\epsilon$ above parameterizes both the small deviation of $\sin\phi_{\epsilon_K} = \Delta M_K/(\Delta M_K^2 + \Delta\Gamma_K^2/4)^{1/2}$ from $1/\sqrt{2}$ and the small contribution from the phase of the isospin-zero $K \to \pi\pi$ decay amplitude. This factor was estimated to $\kappa_\epsilon = 0.92 \pm 0.02$ [21] assuming the standard model. Its modification in the presence of new physics will not alter our analysis, and we will ignore this complication. The mass difference $\Delta M_K$ between the two eigenstates $K_L$ and $K_S$ receives both short-distance and long-distance contributions, such that the constraint on possible new-physics effects in the short-distance part,

$$(\Delta M_K)^{\text{SD}} = 2\,\text{Re}\left(M_{12}^K\right), \tag{4.5}$$



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| | | | |
|---|---|---|---|
| $\Delta M_K^{\rm exp} = (3.483 \pm 0.006) \cdot 10^{-12}$ MeV | [23] | $\lvert\epsilon_K\rvert^{\rm exp} = (2.229 \pm 0.012) \cdot 10^{-3}$ | [23] |
| $\Delta M_d^{\rm exp} = (3.337 \pm 0.033) \cdot 10^{-10}$ MeV | [23] | $S_{J/\psi K_S}^{\rm exp} = 0.671 \pm 0.024$ | [24] |
| $\Delta M_s^{\rm exp} = (117.0 \pm 0.8) \cdot 10^{-10}$ MeV | [23] | $\phi_s^{\rm exp} = (-0.77^{+0.29}_{-0.37}) \cup (-2.36^{+0.37}_{-0.29})$ rad | [24] |

**Table 1.** Current experimental values of the various $\Delta F = 2$ observables considered in eqs. (4.4)–(4.9).

is somewhat diluted among hadronic uncertainties. Despite its precise experimental knowledge, $\Delta M_K$ will thus play a minor role in our study.

On the contrary, when new sources of CP-violation in the kaon system are small, two observables in the $B_d$ system will prove useful to gain information on the mixing angle $\theta$. These are the mass difference,

$$\Delta M_d = 2 \left| M_{12}^{B_d} \right| , \tag{4.6}$$

and the coefficient of the $\sin(\Delta M_d\, t)$ term in the $B_d \to J/\psi K_S$ time-dependent CP asymmetry,

$$S_{J/\psi K_S} = \sin\left(2\beta + \phi_d^\Delta\right) \simeq \mathrm{Im}\left(\frac{M_{12}^{B_d}}{\left|M_{12}^{B_d}\right|}\right), \quad \beta \equiv \arg\left[-\frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}}\right], \quad \phi_d^\Delta \equiv \arg\frac{M_{12}^{B_d}}{M_{12}^{B_d,\,\rm SM}}. \tag{4.7}$$

The phase $\phi_d^\Delta$ parameterizes CP-violating effects beyond the SM in $B_d - \overline{B}_d$ mixing. Here and in the following, we use the standard CKM phase convention.

Finally, we will also consider the mass difference in the $B_s$ system,

$$\Delta M_s = 2 \left| M_{12}^{B_s} \right| , \tag{4.8}$$

as well as the phase measured in the $B_s \to J/\psi\,\phi$ time-dependent angular distribution,

$$-2\beta_s^{\rm eff} = -2\beta_s + \phi_s^\Delta \simeq \arcsin\left(\mathrm{Im}\,\frac{M_{12}^{B_s}}{\left|M_{12}^{B_s}\right|}\right), \quad \beta_s \equiv -\arg\left[-\frac{V_{ts}^* V_{tb}}{V_{cs}^* V_{cb}}\right], \quad \phi_s^\Delta \equiv \arg\frac{M_{12}^{B_s}}{M_{12}^{B_s,\,\rm SM}}. \tag{4.9}$$

In the SM, $\beta_s$ is tiny: $2\beta_s \simeq 0.04$. As long as $\phi_s^\Delta$ is not too small, we thus have $-2\beta_s^{\rm eff} \simeq \phi_s^\Delta$. On the other hand, one also has $\phi_s \equiv \arg(-M_{12}^{B_s}/\Gamma_{12}^{B_s}) \simeq \phi_s^\Delta$ [22]. In the following, we will thus identify $\phi_s = -2\beta_s^{\rm eff}$.

The current experimental values of the various observables above are reported in table 1.

### 4.1 Standard-Model contributions

In the standard model, $W$ box diagrams with virtual $t$ and/or $c$ flavors generate the effective operators

$$Q_K^{\rm VLL} = \left(\overline{d}_L \gamma_\mu s_L\right)\left(\overline{d}_L \gamma^\mu s_L\right), \qquad Q_{B_q}^{\rm VLL} = \left(\overline{q}_L \gamma_\mu b_L\right)\left(\overline{q}_L \gamma^\mu b_L\right) \tag{4.10}$$


**Figure 1.** Dominant short-distance contributions to $M_{12}^K$ (a) in the SM; (b) in the CMM extension.

for kaons (see figure 1a) and $B_q$ ($q = s$ or $d$), respectively. The corresponding Wilson coefficients at the scale $\mu_P$ read

$$\begin{aligned} C_K^{\rm VLL}(\mu_K) &= 4U_K(\mu_K)\Big[(V_{cd}^*V_{cs})^2\eta_1 S_0(x_c) + 2(V_{cd}^*V_{cs})(V_{td}^*V_{ts})\eta_3 S_0(x_c,x_t) \\ &\qquad + (V_{td}^*V_{ts})^2\,\eta_2 S_0(x_t)\Big], \\ C_{B_q}^{\rm VLL}(\mu_{B_q}) &= 4U_{B_q}(\mu_{B_q})(V_{tq}^*V_{tb})^2\eta_B S_0(x_t), \end{aligned} \tag{4.11}$$

where the factors

$$\begin{aligned} U_K(\mu) &= \left[\alpha_s^{(3)}(\mu)\right]^{-2/9}\left[1+\frac{\alpha_s^{(3)}(\mu)}{4\pi}J_3\right] \\ \text{and}\quad U_{B_q}(\mu) &= \left[\alpha_s^{(5)}(\mu)\right]^{-6/23}\left[1+\frac{\alpha_s^{(5)}(\mu)}{4\pi}J_5\right] \end{aligned} \tag{4.12}$$

encode the $\mu_K, \mu_{B_q}$-dependent parts of the short-distance QCD corrections up to next-to-leading order (NLO), while $\eta_i$ account for their $\mu_K, \mu_{B_q}$-independent contributions [25, 26]; their values are given in table 2. The loop functions $S_0(x_q)$ and $S_0(x_c, x_t)$ are listed in the appendix. Finally, $x_q = m_q^2/M_W^2$ and $m_q \equiv \overline{m}_q(m_q)$ is the $\overline{\rm MS}$ mass.

In order to compute $M_{12}^{K,B_q}$, we still need the matrix elements of $Q_K^{\rm VLL}$ and $Q_{B_q}^{\rm VLL}$. These are parameterized in terms of "bag factors" $B_P$, computed at the scale $\mu = \mathcal{O}(\mu_P)$:

$$\left\langle P^0\middle|Q_P^{\rm VLL}(\mu)\middle|\overline{P}^0\right\rangle = \frac{2}{3}M_P^2F_P^2B_P(\mu), \tag{4.13}$$

where $F_P$ is the decay constant of the $P$ meson. The scale dependences of $U_P(\mu)$ and $B_P(\mu)$ cancel each other, so that it is convenient to define the renormalization-group-invariant parameters $\widehat{B}_P = B_P(\mu)U_P(\mu)$. Eqs. (4.11), (4.12), and (4.13) then lead to

$$\begin{aligned} (M_{12}^K)^{\rm SM} &= \frac{G_F^2M_W^2}{12\pi^2}M_KF_K^2\widehat{B}_K\Big[(\lambda_{ds}^c)^2\eta_1 S_0(x_c) + 2(\lambda_{ds}^c)(\lambda_{ds}^t)\eta_3 S_0(x_c,x_t) \\ &\qquad + (\lambda_{ds}^t)^2\,\eta_2 S_0(x_t)\Big], \\ (M_{12}^{B_q})^{\rm SM} &= \frac{G_F^2M_W^2}{12\pi^2}M_{B_q}F_{B_q}^2\widehat{B}_{B_q}(\lambda_{qb}^t)^2\eta_B S_0(x_t), \end{aligned} \tag{4.14}$$

where one defines $\lambda_{ij}^k = V_{ki}^*V_{kj}$.



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### 4.2 CMM contributions

In the context of the CMM model, the dominant supersymmetric effects originate from gluino box diagrams with virtual $\tilde{d}_R$, $\tilde{s}_R$, and $\tilde{b}_R$ flavors due to the large mixings in eq. (3.8). This gives rise to the parity-reflected operators (figure 1b)

$$Q_K^{\rm VRR} = \left(\overline{d}_R\gamma_\mu s_R\right)\left(\overline{d}_R\gamma^\mu s_R\right), \qquad Q_{B_q}^{\rm VRR} = \left(\overline{q}_R\gamma_\mu b_R\right)\left(\overline{q}_R\gamma^\mu b_R\right). \tag{4.15}$$

The initial conditions for the Wilson coefficients at the SUSY scale $M_S = \mathcal{O}(m_{\tilde{d}_j}, m_{\tilde{g}})$ read [9, 11]

$$C_{K,B_q}^{\rm CMM}(M_S) = \frac{16\pi^2}{G_F^2 M_W^2}\frac{\alpha_s^2(M_S)}{2m_{\tilde{g}}^2}\sum_{k,m=1}^{3}(R_d)_{mj}(R_d)_{mi}^*(R_d)_{kj}(R_d)_{ki}^* L_0(r_m, r_k), \tag{4.16}$$

where $(i,j) = (1,2)$ in the kaon case, $(1,3)$ in the $B_d$ case, and $(2,3)$ in the $B_s$ case. The loop function $L_0(r_m, r_k)$ is defined in the appendix, the down-type squark mixing matrix $R_d$ was given in eq. (3.8), and $r_j = m_{\tilde{d}_j}^2/m_{\tilde{g}}^2$. Exploiting the mass degeneracy of the first two generations (see eq. (3.9)) as well as the unitarity of $R_d$, eq. (4.16) simplifies to

$$\begin{aligned} C_{K,B_q}^{\rm CMM}(M_S) = {} & \frac{16\pi^2}{G_F^2 M_W^2}\frac{\alpha_s^2(M_S)}{2m_{\tilde{g}}^2}\left[(R_d)_{3j}(R_d)_{3i}^*\right]^2 \\ & \times\left\{L_0(r_1, r_1) - 2L_0(r_1, r_3) + L_0(r_3, r_3)\right\}, \end{aligned} \tag{4.17}$$

$$r_1 = m_{\tilde{d}}^2/m_{\tilde{g}}^2, \qquad r_3 = m_{\tilde{d}}^2\left(1-\Delta_{\tilde{d}}\right)/m_{\tilde{g}}^2. \tag{4.18}$$

The RGE of the above Wilson coefficients from the scale $M_S$ down to the scale $\mu_{K,B_q}$ is performed in two steps: first, the leading-order matching coefficients in eq. (4.17) are evolved down to $\mu_t = \mathcal{O}(m_t)$ by means of the leading-order RGE factor $\eta_6 = [\alpha_s^{(6)}(M_S)/\alpha_s^{(6)}(\mu_t)]^{2/7}$. The remaining evolution, running over two orders of magnitude, is achieved using NLO formulas — essentially the $U_K(\mu_K)$, $\eta_2$, $U_{B_q}(\mu_{B_q})$, and $\eta_B$ factors of section 4.1. The $\mathcal{O}(\alpha_s)$ QCD corrections to the SM function $S_0(x_t)$ at the scale $\mu_t$, which are contained in $\eta_2$ and $\eta_B$, should be removed. Denoting them by $r = 0.985$ [26], we get

$$C_K^{\rm CMM}(\mu_K) = U_K(\mu_K)\,\eta_2\,\frac{1}{r}\,\eta_6\,C_K^{\rm CMM}(M_S), \tag{4.19}$$

and similarly for $C_{B_q}^{\rm CMM}(\mu_{B_q})$. The cancellation of the $\mu_t$-dependence between the two parts of the evolution is of course incomplete, yet this is a numerically small effect which can be neglected.

The bag parameters of the effective operators $Q_K^{\rm VRR}$ and $Q_{B_q}^{\rm VRR}$ are identical to those of the SM operators in eq. (4.13) such that the CMM contributions to the matrix elements $M_{12}^P$ finally read

$$\begin{aligned} (M_{12}^K)^{\rm CMM} &= \frac{\alpha_s^2(M_S)}{6m_{\tilde{g}}^2} M_K F_K^2 \widehat{B}_K \frac{e^{-2i\phi_K}\sin^2(2\theta)}{16}\frac{\eta_2\eta_6}{r}\,S^{(\tilde{g})}(r_1, r_3), \\ (M_{12}^{B_d})^{\rm CMM} &= \frac{\alpha_s^2(M_S)}{6m_{\tilde{g}}^2} M_{B_d} F_{B_d}^2 \widehat{B}_{B_d} \frac{e^{-2i\phi_{B_d}}\sin^2\theta}{4}\frac{\eta_B\eta_6}{r}\,S^{(\tilde{g})}(r_1, r_3), \\ (M_{12}^{B_s})^{\rm CMM} &= \frac{\alpha_s^2(M_S)}{6m_{\tilde{g}}^2} M_{B_s} F_{B_s}^2 \widehat{B}_{B_s} \frac{e^{-2i\phi_{B_s}}\cos^2\theta}{4}\frac{\eta_B\eta_6}{r}\,S^{(\tilde{g})}(r_1, r_3), \end{aligned} \tag{4.20}$$

| | | | |
|---|---|---|---|
| $\kappa_\epsilon = 0.92 \pm 0.02$ | [21] | $\lvert V_{us}\rvert = 0.2246 \pm 0.0012$ | [27] |
| $F_K = (156.1 \pm 0.8)\,\text{MeV}$ | [27] | $\lvert V_{cb}\rvert = (41.6 \pm 0.6) \cdot 10^{-3}$ | [23] |
| $\widehat{B}_K = 0.75 \pm 0.07$ | [28] | $\lvert V_{ub}\rvert = (3.95 \pm 0.35) \cdot 10^{-3}$ | [23] |
| $F_{B_s}\widehat{B}^{1/2}_{B_s} = (270 \pm 30)\,\text{MeV}$ | [28] | $\gamma = (70.7^{+\,5.7}_{-\,7.0})^\circ$ | [see text] |
| $\xi \equiv \frac{F_{B_s}\widehat{B}^{1/2}_{B_s}}{F_{B_d}\widehat{B}^{1/2}_{B_d}} = (1.21 \pm 0.04)$ | [28] | $\eta_1 = (1.32 \pm 0.32)\left[\frac{1.30\,\text{GeV}}{\overline{m}_c(m_c)}\right]^{1.1}$ | [25, 29] |
| $\overline{m}_c(m_c) = (1.266 \pm 0.014)\,\text{GeV}$ | [32] | $\eta_2 = 0.57 \pm 0.01$ | [26, 29] |
| $\overline{m}_t(m_t) = (162.1 \pm 1.2)\,\text{GeV}$ | [31, 33] | $\eta_3 = 0.47 \pm 0.05$ | [25, 29] |
| $\alpha_s(M_Z) = 0.1176 \pm 0.0020$ | [23] | $\eta_B = 0.551 \pm 0.007$ | [26, 30] |

**Table 2.** Input parameters.

where we explicitly display the factors $(R_d)_{3i}$ in eq. (4.17), $S^{(\tilde{g})}(r_1, r_3) = L_0(r_1, r_1) - 2L_0(r_1, r_3) + L_0(r_3, r_3)$, and the CMM phases $\phi_K$, $\phi_{B_d}$, and $\phi_{B_s}$ have been defined in eq. (3.10). Note that they fulfill the relation $\phi_{B_d} = \phi_K + \phi_{B_s}$.

### 4.3 Additional supersymmetric contributions

Finally, we comment on the supersymmetric contributions which do not exhibit the large enhancement factors characteristic of the CMM model, namely charged-Higgs($H$)-quark and chargino($\chi$)-squark box diagrams. They do not introduce new operators, and the flavor structure of the corresponding matrix elements is the same as in the SM,

$$\begin{aligned}(M_{12}^K)^{H+\chi} &= \frac{G_F^2 M_W^2}{12\pi^2} M_K F_K^2 \widehat{B}_K \\ &\quad \times \left\{2(\lambda^c_{ds})(\lambda^t_{ds})\eta_3^H S_H(c,t) + (\lambda^t_{ds})^2\,\eta_2\,[S_H(t,t) + S_\chi(t,t)]\right\}, \\ (M_{12}^{B_q})^{H+\chi} &= \frac{G_F^2 M_W^2}{12\pi^2} M_{B_q} F_{B_q}^2 \widehat{B}_{B_q} (\lambda^t_{qb})^2 \eta_B\,[S_H(t,t) + S_\chi(t,t)]\,.\end{aligned} \tag{4.21}$$

The loop functions $S_H(c,t)$, $S_H(t,t)$, and $S_\chi(t,t)$ are given explicitly in ref. [7]. The factor $\eta_3^H = 0.21$ [7] denotes leading-order QCD corrections to the charged-Higgs box with virtual flavors $(c,t)$. Numerically, charged-Higgs and chargino contributions are small compared to CMM effects. We checked explicitly that they can be neglected in our analysis.

## 5 Numerical analysis

We are now ready to investigate the constraints of $K - \overline{K}$ and $B_d - \overline{B}_d$ mixing on the angle $\theta$ in the down-type squark mixing matrix $R_d$. Since we do not expect a miraculous cancellation of the phases $\phi_1$ and $\phi_2$, we will first focus on the case where $\sin 2\phi_K \sim \mathcal{O}(1)$ (section 5.2) and derive constraints on $\theta$ from $|\epsilon_K|$ alone. We will then turn to the special case $\sin 2\phi_K \sim 0$ (section 5.3) where, as we will see, interesting constraints can still be obtained from $\Delta M_K$, $\Delta M_d$, $S_{J/\psi K_S}$, and $\Delta M_d/\Delta M_s$.

The values of the various input parameters adopted in our numerical analysis are reported in table 2. Inputs related to CKM elements have to be protected from new-physics impact. To this end, we determine the CKM matrix from the elements $|V_{ub}|$, $|V_{cb}|$,



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$|V_{us}|$, and $\delta$, the CP-phase in the standard parametrization, which equals the angle $\gamma$ of the unitarity triangle to very good accuracy. The three CKM elements are extracted from tree-level decays. We use $|V_{us}| = 0.2246 \pm 0.0012$ [27], the inclusive determination $|V_{cb}| = (41.6 \pm 0.6) \cdot 10^{-3}$ [23], and the average of inclusive and exclusive determinations $|V_{ub}| = (3.95 \pm 0.35) \cdot 10^{-3}$ [23]. The angle $\gamma$ is determined via $\gamma = \pi - \alpha - \beta = \pi - \alpha^{\text{eff}} - \beta^{\text{eff}}$, with $\beta^{\text{eff}} = \beta + \phi_d^\Delta/2 = (21.1 \pm 0.9)^\circ$ from $S_{J/\psi K_S}$ [24] and $\alpha^{\text{eff}} = \alpha - \phi_d^\Delta/2 = (88.2\,^{+\,6.1}_{-\,4.8})^\circ$ from $B \to \pi\pi,\ \pi\rho,\ \rho\rho$ decays [34]. The dependence on the new-physics phase $\phi_d^\Delta$ cancels out in the sum $\alpha^{\text{eff}} + \beta^{\text{eff}}$, such that $\gamma = (70.7\,^{+\,5.7}_{-\,7.0})^\circ$ is indeed free from new-physics contamination.

No assumption is made on the squark mixing parameters $\theta$, $\phi_K$, $\phi_{B_d}$, and $\phi_{B_s}$ prior to the analysis of the observables in table 1. The supersymmetric parameters (in particular $m_{\tilde{g}}$, $r_1$, and $r_3$, or equivalently $m_{\tilde{g}}$, $m_{\tilde{d}}$, and $\Delta_{\tilde{d}}$), on the other hand, are chosen such as to satisfy the constraints coming from other observables. The identification of viable sets of SUSY parameters is the subject of the next section.

### 5.1 CMM parameter sets

In the CMM model, the large number of free SUSY parameters shrinks to six input parameters at the electroweak scale (in addition to $\theta$ and the CMM phases $\phi_K$, $\phi_{B_d}$, and $\phi_{B_s}$). These can be chosen as the gluino mass $m_{\tilde{g}}$, the first-generation $\tilde{d}_R$ and $\tilde{u}_R$ soft masses $m_{\tilde{d}}$ and $m_{\tilde{u}}$,[7] the ratio of the (11)-elements of the trilinear and Yukawa couplings in the super-CKM basis $a_d^1 = (A_d)_{11}/(Y_d)_{11}$, the phase of the $\mu$ parameter in the Higgs potential $\arg(\mu)$, and the ratio of the two Higgs-doublet vevs $\tan\beta$. The RGE links these CMM inputs to the remaining SUSY parameters via the assumption of universal soft-breaking parameters at the Planck scale and the intermediate SO(10) and SU(5) GUT relations. Note that the similar input parameters in the CMM model and in specific SUSY scenarios without grand unification still lead to very different phenomenologies. In such well-studied scenarios as mSUGRA or the CMSSM, the SUSY-breaking parameters are universal at $M_{\text{GUT}}$, as mentioned in the Introduction, leaving the universal gaugino and scalar masses, $m_{1/2}$ and $m_0$, the trilinear coupling $A$, as well as the sign of $\mu$ and $\tan\beta$ as free parameters. In contrast to GUT models, however, these scenarios do not relate quarks and leptons to each other; the MSSM fields can be rotated independently and the large lepton mixing angles do not become visible in the quark sector.

To establish benchmarks for our analysis of the down-squark mixing angle $\theta$ in $K - \overline{K}$ and $B_d - \overline{B}_d$ mixing, we make sure that the chosen CMM input parameters are in accord with the other observables sensitive to CMM effects, and that they respect constraints common to generic SUSY scenarios. To this end, we make use of the *Mathematica* code written by the authors of ref. [12], which implements the relations between the CMM input parameters discussed above and the remaining SUSY parameters at the electroweak scale. The most restrictive observable is the experimental lower bound on the mass of the lightest Higgs boson $m_h$. For small values of $\tan\beta$ it is close to the SM bound, $m_h \geq 114.4\,\text{GeV}$ [36]. The main radiative corrections to the tree-level Higgs mass in the MSSM, $m_h^{\text{tree}} \leq M_Z |\cos 2\beta|$, stem from (s)top loops. For very small values of $\tan\beta \approx 3$ the

[7] The specification of both $m_{\tilde{d}}$ and $m_{\tilde{u}}$ fixes the D-term scalar mass splitting [12, 35].



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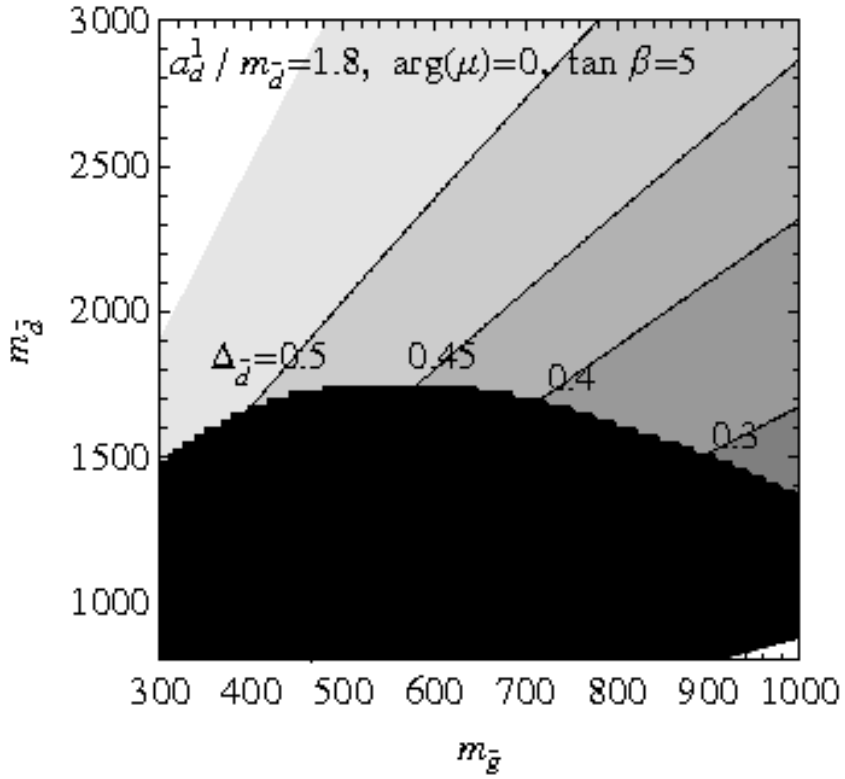


**Figure 2.** Down-squark mass splitting $\Delta_{\tilde{d}}$ as a function of $m_{\tilde{g}}$ and $m_{\tilde{d}}$ [GeV]. White: negative soft masses. Black: excluded by lower bound on light Higgs mass.

large top Yukawa coupling in the RGE drives the stop mass to low values, such that the Higgs mass bound cannot be fulfilled. In our analysis we choose $\tan\beta = 5$, such that the top Yukawa coupling gets smaller, but the natural hierarchy between the top and bottom Yukawa couplings, induced by $v_0/M_{\rm Pl}$ in the CMM superpotential, is preserved. We fix the inputs $a_d^1/m_{\tilde{d}} = 1.8$ and $\arg(\mu) = 0$, such that the allowed space for $m_{\tilde{g}}$ and $m_{\tilde{d}}$ around 1 TeV is large. Finally, we take $m_{\tilde{u}} = m_{\tilde{d}}$ as in ref. [12]. In figure 2 we show the mass splitting parameter $\Delta_{\tilde{d}}$ in the $m_{\tilde{g}} - m_{\tilde{d}}$ plane for this scenario. Black regions are excluded by the Higgs mass bound. White regions are forbidden due to negative soft mass parameters. Additional constraints arise from processes reflecting the large atmospheric neutrino mixing angle like $\tau \to \mu\gamma$, $b \to s\gamma$, and the mass difference $\Delta M_s$; these can cut further into the low $m_{\tilde{g}}$ and $m_{\tilde{d}}$ regions.

Based on these considerations, we select three sets of CMM input parameters, given in table 3. As said above, these parameters are defined at the electroweak scale, more precisely at $M_Z$, in ref. [12]. For consistency, we will thus set $M_S = M_Z$ (and correspondingly $\eta_6 = 1$, neglecting the small effect of $m_t \neq M_Z$) in our analysis of meson-antimeson mixing. Sets 2 and 3 do satisfy the $\Delta M_s$ constraint for all values of $\theta$ and $\phi_{B_s}$, while Set 1 requires $|2\phi_{B_s}|$ to be between 1.2 and 2.4 radians for small $\theta$ to satisfy this constraint. Note that especially Set 1 (with small $m_{\tilde{g}}$ and large $\Delta_{\tilde{d}}$) is chosen such that CMM effects in $b \to s$, $b \to d$, and $s \to d$ transitions are large.



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| | $m_{\tilde{g}}$ [GeV] | $m_{\tilde{d}}$ [GeV] | $\Delta_{\tilde{d}}$ | $\theta^{\max}$ [°] |
|---|---|---|---|---|
| Set 1 | 400 | 2000 | 0.52 | 0.5 |
| Set 2 | 700 | 2000 | 0.44 | 0.9 |
| Set 3 | 700 | 3000 | 0.51 | 0.9 |

**Table 3.** CMM parameter sets for fixed $a_d^1/m_{\tilde{d}} = 1.8$, $\arg(\mu) = 0$, and $\tan\beta = 5$, satisfying the constraints discussed in section 5.1. The last column shows the maximal mixing angle $\theta^{\max}$ allowed by $|\epsilon_K|$ for $\sin 2\phi_K = 1$ (the symmetric solution $\theta \in [(\pi - \theta^{\max})/2, \pi/2]$ is excluded by $B$ physics observables, see figure 4).

### 5.2 Scenario I: $\sin 2\phi_K \sim \mathcal{O}(1)$

As long as the CMM phase $\phi_K$ is not too close to zero, $|\epsilon_K|$ gives the best constraint on $\theta$. The dependence of $\theta^{\max}$ on the relevant combinations of parameters, i.e., $\sin 2\phi_K/m_{\tilde{g}}^2$, $m_{\tilde{d}}/m_{\tilde{g}}$, and $\Delta_{\tilde{d}}$, is summarized in figure 3-left. The plain black and dashed gray lines (which happen to be nearly superposed) correspond to $m_{\tilde{d}}/m_{\tilde{g}}$ and $\Delta_{\tilde{d}}$ of Set 2 and Set 1, respectively, while the two other lines are obtained by interchanging $m_{\tilde{d}}/m_{\tilde{g}}$. As one can see, for $|\sin 2\phi_K|/m_{\tilde{g}}^2 \gtrsim 1\,\text{TeV}^{-2}$ and typical values of the parameters $m_{\tilde{d}}/m_{\tilde{g}}$ and $\Delta_{\tilde{d}}$, $\theta^{\max}$ is of the order of one degree. Figure 3 has been obtained treating the errors in table 2 as flat, yet a different error treatment — and/or inflated errors in table 2 — would not change this picture significantly. Fixing $\phi_K$ to $\pi/4$, the precise limits obtained for the various parameter sets defined in section 5.1 are displayed in the last column of table 3. The small contributions in section 4.3 have no impact on these numbers.

In the $B_d$ and $B_s$ systems, the SM contributions are not as suppressed as for $\epsilon_K$. Consequently, the smallness of $\theta^{\max}$ prevents any visible effect in $\Delta M_d$ and $S_{J/\psi K_S}$, while the formulas for $\Delta M_s$ and $\phi_s$ are well approximated setting $\theta = 0$. Interestingly, sizeable CMM contributions in the $B_s$ system may be welcome to reduce the $2.2\sigma$ discrepancy between the SM prediction for $\phi_s$ and its experimental value [24]. Within Set 1 it is possible to bring this discrepancy down to the one-sigma level while satisfying all existing constraints, see figure 3-right.

Finally, we briefly comment on the dependence of $\theta^{\max}$ on the hypothesis of tribimaximal lepton mixing. In particular, one might expect the 23-mixing angle to be large but not $\pi/4$. In this case, $\text{Im}\,[(R_d)_{32}(R_d)_{31}^*]^2 = -\frac{1}{4}\sin^4\theta_{23}\sin^2(2\theta)\,\sin(2\phi_K)$ for $\theta_{13} = 0$. Hence, for large $\theta_{23}$, the constraints on $\theta$ do not differ much. For a sizeable 13-mixing angle in $V_\ell$, $|\epsilon_K|$ gets additional contributions:

$$\begin{aligned}\Delta\left(\text{Im}[(R_d)_{32}(R_d)_{31}^*]^2\right) =& \sin\theta_{13}\sin^3\theta_{23}\sin(2\theta)[\sin(2\phi_K)\cos(2\theta)\cos(\phi_3-\phi_2+\alpha_4-\alpha_1-\delta) \\ & - \cos(2\phi_K)\sin(\phi_3-\phi_2+\alpha_4-\alpha_1-\delta)] + \mathcal{O}\left(\sin^2\theta_{13}\right).\end{aligned} \tag{5.1}$$

No large numerical factors offset the $\sin\theta_{13}$-suppression, so that the modified $\theta$ bounds are again as stringent as those exemplified in figure 3.

Up to now, we have taken the viewpoint of a fixed sparticle spectrum, and investigated the correlation between effects in $b \to s$ and $b, s \to d$ transitions governed by the mixing angle $\theta$. As $\theta$ turns out to be restricted to very small values, it is interesting to consider



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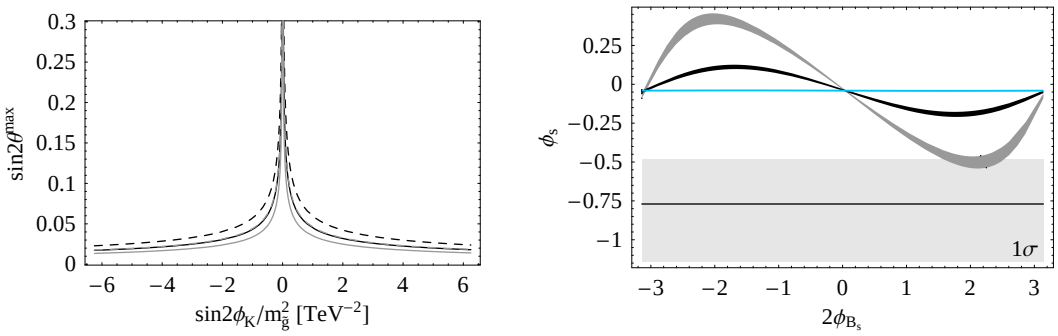


**Figure 3.** Left: Constraints on $\theta$ from $|\epsilon_K|$ as a function of $\sin 2\phi_K/m_{\tilde{g}}^2$ for $\Delta_{\tilde{d}} = 0.44$ (black) / 0.52 (gray) and $m_{\tilde{d}}/m_{\tilde{g}} = 2.86$ (plain) / 5 (dashed). Right: CP-violating phase $\phi_s$ as a function of the CMM phase $\phi_{B_s}$. The light gray (dark gray) curve corresponds to Set 1 (Set 2) with $\theta = 0$. The SM prediction (horizontal line) is recovered for $2\phi_{B_s} = 0, \pm\pi$. The broad gray band indicates the one-sigma measurement [24].

the opposite viewpoint of a fixed 'natural' $\theta$ value — say, $\sin\theta = 0.5$ — and derive the corresponding constraints on sparticle masses from $\epsilon_K$. Setting again $\phi_K$ to $\pi/4$, we find that a soft mass scale $m_{\tilde{g}} \simeq 2\,\text{TeV}$ is possible only if the ratio $m_{\tilde{d}}/m_{\tilde{g}} \simeq 1$. In such a scenario, however, the mass splitting parameter $\Delta_{\tilde{d}}$ is very small (cf. figure 2), such that CMM effects in other observables are negligible. For larger values of the ratio $m_{\tilde{d}}/m_{\tilde{g}}$, $\Delta_{\tilde{d}}$ increases and accordingly the constraints on $m_{\tilde{g}}$ are much more stringent (for example $m_{\tilde{g}} \gtrsim 20\,\text{TeV}$ for $m_{\tilde{d}}/m_{\tilde{g}} = 2$). CMM effects in $B_d$ and $B_s$ physics are thus again killed, this time by the strong $1/m_{\tilde{g}}^2$ suppression factor.

### 5.3 Scenario II: $\sin 2\phi_K \sim 0$

If $\sin 2\phi_K$ is close to zero, CMM effects cannot make their way into $\text{Im}M_{12}^K$ anymore, and the best constraints on $\theta$ are obtained from $\Delta M_K$ and $B$ physics observables. As mentioned in section 4, $\Delta M_K$ is plagued by hadronic uncertainties, so that we merely impose $|\Delta M_K^{\text{CMM}}| < \Delta M_K^{\text{exp}}$ to stay on the conservative side. In this case, for $m_{\tilde{g}} \simeq 700\,\text{GeV}$, the constraint from $\Delta M_K$ only starts to compete with that from $|\epsilon_K|$ when $|\phi_K| = \mathcal{O}(0.1^\circ)$, corresponding to $\theta^{\max} \simeq 10^\circ - 30^\circ$ (depending on the precise values of $\Delta_{\tilde{d}}$ and $m_{\tilde{d}}/m_{\tilde{g}}$). The constraints from $\Delta M_d$, $S_{J/\psi K_S}$, and $\Delta M_d/\Delta M_s$ are in general better, as we illustrate in figure 4 for Set 1 and Set 2. Note that the constraint from $\Delta M_d/\Delta M_s$ depends on both $\phi_{B_s}$ and $\phi_{B_d} = \phi_K + \phi_{B_s}$. The plots shown in figure 4 correspond to $\phi_K = 0$ and $\phi_K = \pi/2$. Other $\phi_K$ values lead to different plots, with however the same general appearance, in particular the exclusion of small $\theta$ angles for some specific $\phi_{B_s}$ values. For these specific values, the tight bounds on $\theta$ derived in section 5.2 are thus even surpassed.

As mentioned previously, $\phi_s$ can cut further into the parameter space, especially for negative $\phi_{B_s}$ values, see figure 3-right. However, this does not change the typical value of $\theta^{\max}$ obtained from $B$ physics observables, which is of ten or a few tens of degrees.



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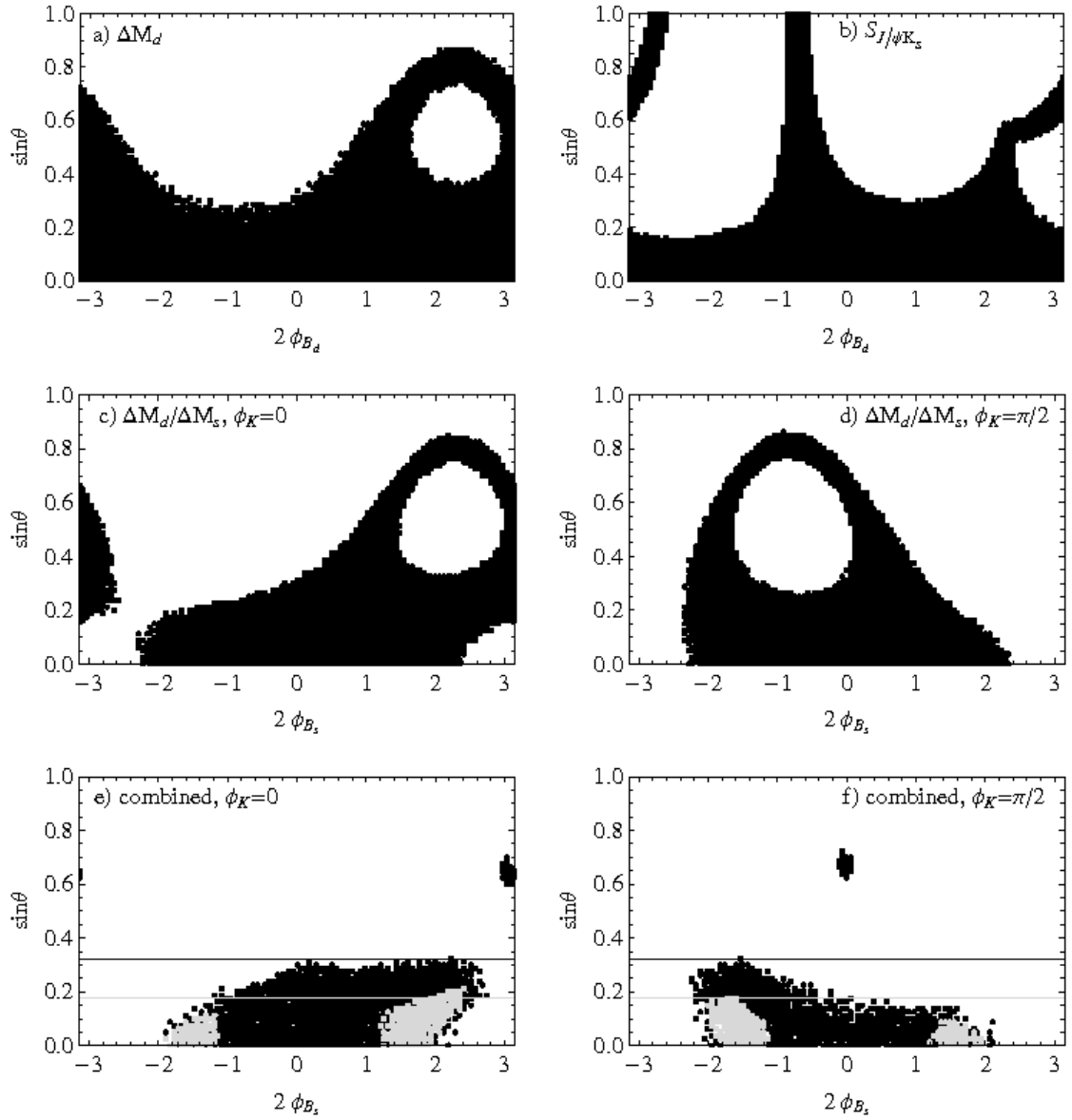


**Figure 4.** Constraints on $\theta$ from $B$ physics observables. Black (gray) points indicate allowed regions in Set 2 (Set 1) parameter space. The first four plots show individual three-sigma constraints from (a) $\Delta M_d$, (b) $S_{J/\psi K_S}$, (c) $\Delta M_d/\Delta M_s$ setting $\phi_K = 0$, (d) $\Delta M_d/\Delta M_s$ setting $\phi_K = \pi/2$. Plots (e) and (f) show the combined (a,b,c) and (a,b,d) constraints, respectively. In the case of Set 1, the three-sigma constraint from $\Delta M_s$ has also been imposed, excluding points outside the $1.2 \lesssim |2\phi_{B_s}| \lesssim 2.4$ range (recall that Set 2 is not affected by this constraint). Imposing further the constraint from $\phi_s$ would remove the gray points with $2\phi_{B_s} < 0$ and the black points with $-1.9 \lesssim 2\phi_{B_s} \lesssim -1.5$ for $\sin\theta$ below 0.15, see figure 3-right. Finally, Set 2 (Set 1) points above the black (gray) horizontal line are excluded by $\Delta M_K$.

### 5.4 Closing the unitarity triangle

Recently, several studies pointed out a possible tension in the SM between the value of $\sin 2\beta$ predicted from $|\epsilon_K|$ and $\Delta M_s/\Delta M_d$, and its direct measurement from $S_{J/\psi K_S}$ [21, 37–39].



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In this section, we illustrate how CMM effects can remove this tension, and simultaneously account for a sizeable CP-violating phase in the $B_s$ system.

Due to the particular sensitivity of $|\epsilon_K|$ to new-physics effects, either $\theta$ or $\phi_K$ must be very small. We will thus consider the two limits $\theta = 0$ and $\phi_K = 0$. For each case, we will compare the value of $\sin 2\beta$ extracted from $S_{J/\psi K_S}$ with its determination from $|V_{us}|$, $|V_{cb}|$, $|\epsilon_K|$, and $\Delta M_s/\Delta M_d$, obtained inverting the following expressions with respect to $\sin 2\beta$ and $R_t$:

$$|\epsilon_K| = \kappa_\epsilon \frac{M_K F_K^2 \widehat{B}_K}{12\sqrt{2}\Delta M_K} \times \left\{ \frac{G_F^2 M_W^2}{\pi^2} |V_{cb}|^2 |V_{us}|^2 \left[ |V_{cb}|^2 R_t^2 \sin(2\beta)\eta_2 S_0(x_t) \right.\right. \tag{5.2}$$
$$\left.\left. + 2R_t \sin\beta (\eta_3 S_0(x_c, x_t) - \eta_1 S_0(x_c)) \right] - \frac{\alpha_s^2(M_S)}{8 m_{\tilde{g}}^2} \sin(2\phi_K) \sin^2(2\theta) \frac{\eta_2 \eta_6}{r} S^{(\tilde{g})}(r_1, r_3) \right\},$$

$$\frac{\Delta M_s}{\Delta M_d} = \xi^2 \frac{M_{B_s}}{M_{B_d}} \tag{5.3}$$
$$\times \frac{\sqrt{(k_1 + X\cos 2\phi_{B_s}\cos^2\theta)^2 + (-2k_2 R_t \sin\beta |V_{us}|^2 - X \sin 2\phi_{B_s}\cos^2\theta)^2}}{\sqrt{\left(R_t^2 \cos 2\beta |V_{us}|^2 + X\cos 2\phi_{B_d}\sin^2\theta\right)^2 + \left(R_t^2 \sin 2\beta |V_{us}|^2 - X \sin 2\phi_{B_d}\sin^2\theta\right)^2}}.$$

Here $k_1 = 1 + |V_{us}|^2(1 - 2R_t\cos\beta)$, $k_2 = 1 + |V_{us}|^2(1 - R_t\cos\beta)$,

$$X = \frac{\pi^2 \alpha_s^2(M_S)\,\eta_6\, S^{(\tilde{g})}(r_1, r_3)}{2|V_{cb}|^2 G_F^2 M_W^2 m_{\tilde{g}}^2\, r\, S_0(x_t)}, \tag{5.4}$$

and $R_t = |V_{td}V_{tb}^*|/|V_{cd}V_{cb}^*|$ is a side of the unitarity triangle (UT). The above expressions hold to 0.5% accuracy. In the SM, this leads to $\sin(2\beta^{\,\epsilon_K}) = 0.81^{\,+0.11}_{\,-0.09}$ with the inclusive $|V_{cb}|$ determination of table 2, and to $\sin(2\beta^{\,\epsilon_K}) = 0.98^{\,+0.02}_{\,-0.11}$ if the exclusive determination from $B \to D^*\ell\nu$ decays, $|V_{cb}|^{\rm excl} = (38.8 \pm 1.1)\cdot 10^{-3}$ [24], is used instead. Note that $|V_{cb}|^{\rm incl}$ does not lead to any significant deviation with respect to $S^{\rm exp}_{J/\psi K_S}$, while a tension is indeed observed with the smaller value $|V_{cb}|^{\rm excl}$. In order to illustrate how CMM effects can compensate for a low $|V_{cb}|$ input in UT analyses, we will adopt the averaged value of ref. [39], $|V_{cb}|^{LS} = (41.0 \pm 0.63)\cdot 10^{-3}$. In the following, we use the CMM input parameters of Set 1. All errors are treated as gaussian.

**$\theta = 0$: CMM effects in $R_t$.** Since for $\theta = 0$ there are no effects in $K$ and $B_d$ mixing, CMM contributions enter the UT only via $\Delta M_s$. From figure 5-left, one sees that $R_t$ has to increase in order to close the UT. This requires a CP-violating phase $2\phi_{B_s} \in [1.2, 1.8]$, taking into account the three-sigma constraints on $\phi_{B_s}$ from $\Delta M_s$ and $\phi_s$. The dashed red curve shows $R_t$ for $\phi_{B_s} = 0.7$, such that the UT determined from $|\epsilon_K|$ and $\Delta M_s/\Delta M_d$ agrees with the $\sin 2\beta$ measurement from $S_{J/\psi K_S}$.

**$\phi_K = 0$, $\theta = 0.1$: CMM effects in $R_t$ and $\beta$.** In this second case, CMM effects affect both $\Delta M_s/\Delta M_d$ and $S_{J/\psi K_S}$. For a fixed angle $\theta$, the UT can be closed by adapting the CMM phase $\phi_{B_s} = \phi_{B_d}$. The resulting apex of the UT is shown by the intersection of the dashed red lines in figure 5-right for $\theta = 0.1$ and $\phi_{B_s} = 0.7$. For any value of $\theta$ allowed



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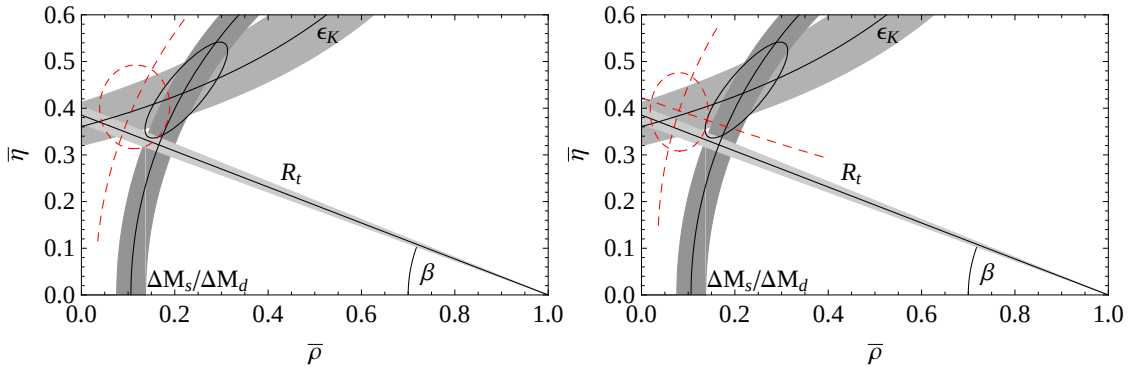


**Figure 5.** One-sigma constraints on the UT from $S_{J/\psi K_S}$ (light gray), $|\epsilon_K|$ (gray), and $\Delta M_s/\Delta M_d$ (dark gray) in the SM. The one-sigma region determined from $|V_{us}|$, $|V_{cb}|$, $|\epsilon_K|$, and $\Delta M_s/\Delta M_d$ assuming the SM is shown in black, and its shift due to CMM effects is indicated in dashed red. Left: Scenario I, $\theta = 0, \phi_{B_s} = 0.7$. Right: Scenario II, $\theta = 0.1, \phi_{B_s} = \phi_{B_d} = 0.7$. CMM inputs: Set 1.

by the constraints from $B_d$ and $B_s$ observables in section 5.3, one can find a phase $\phi_{B_s}$ to close the UT.

Deviations from these two limit cases, i.e., small but nonzero $\theta$ or $\phi_K$ values, rapidly generate CMM effects in $|\epsilon_K|$ as well (figure 3-left). These can lower the band from the $|\epsilon_K|$ constraint in the $(\overline{\rho}, \overline{\eta})$ plane, directly making up for the low $|V_{cb}|$ input value.

## 6 Conclusions

Grand-unified theories introduce relations among quark and lepton masses and mixings. Motivated by the large atmospheric mixing angle in the neutrino sector, several studies focussed on the consequences of the SU(5) Yukawa relation $\mathsf{Y}_d = \mathsf{Y}_e^\top$ in $b \to s$ transitions. In this work, we considered corrections to this relation which are essential to account for the observed light quark and lepton masses. In particular, we investigated the effects on $s \to d$ and $b \to d$ transitions of the additional rotation of the $d_R$ and $s_R$ quarks. This deviation with respect to the PMNS matrix, denoted by $U$, can be parameterized by an additional mixing angle $\theta$ (see eqs. (3.4), (3.5)).

In our analysis, we focussed on models with small Higgs representations; a modified version of the CMM model served as our specific scenario. In this setup, the differences between the down-quark and charged-lepton masses are naturally explained by dimension-five Yukawa operators. The associated supplementary rotation $\theta$ was constrained from $K - \overline{K}$ and $B_d - \overline{B}_d$ mixing observables. In particular, we found that, in the absence of fortuitous cancellations among the new phases in the matrix $U$, $|\epsilon_K|$ sets a stringent bound on $\theta$, $\theta^{\max} \sim \mathcal{O}(1^\circ)$. Consequently, in the basis where the charged-lepton Yukawa couplings are diagonal, the matrix $\mathsf{D}_e\widetilde{\mathsf{Y}}_\sigma + \widetilde{\mathsf{Y}}_\sigma^\dagger\mathsf{D}_e + 5\frac{\sigma}{v_0}\widetilde{\mathsf{Y}}_\sigma^\dagger\widetilde{\mathsf{Y}}_\sigma$ (in the notations of eqs. (2.5), (3.3)) must be diagonal as well. Barring cancellations, this implies that the flavor structure of the couplings which modify the Yukawa unification must be similar to that of the initial terms. In other words, in the corrected relation $\mathsf{Y}_d = \mathsf{Y}_e^\top + 5\frac{\sigma}{v_0}\mathsf{Y}_\sigma$ (eq. (3.3)), the



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three matrices $\mathsf{Y}_\sigma$, $\mathsf{Y}_d$, and $\mathsf{Y}_e^\top$ must be essentially aligned. Constraints from $B$ physics observables ($\Delta M_d$, $S_{J/\psi K_S}$, and $\Delta M_s/\Delta M_d$) were also analyzed, and shown to imply the looser bound $\theta^{\max} \sim \mathcal{O}(10^\circ)$.

While we have worked out this analysis for a specific GUT model, our results hold in general for models with small Higgs representations: large effects of the neutrino mixing angles on $b_R \to s_R$ transitions lead to large effects in $b_R \to d_R$ and $s_R \to d_R$ transitions for natural values of the parameters, once the mass relations for the light quarks and leptons are corrected. An efficient mechanism is naturally needed to render the mixing among right-handed $d$-quarks visible. In the CMM model, this mechanism is provided by the fast SO(10) running of the $\tilde{d}_R$ soft mass matrix, which generates the large universality breaking $\Delta_{\tilde{d}}$ at the electroweak scale. Of course, other GUT scenarios could include additional sources of flavor and CP violation inducing effects in $|\epsilon_K|$. These could soften the constraints on $\theta$. Yet they would have to be fairly fine-tuned to cancel the potentially large impact of the corrections from the $d_R$ rotation matrix $R_d$ (eq. (3.4)).

Interestingly, the correction operators which are of importance for proton decay but contribute equally to the fermion masses ought to have a different flavor structure in order to be in agreement with the experimental limit [17]. Both types of operators are generically present in GUTs. Hence, our analysis is an important step in establishing a consistent grand-unified model.

Finally, we also considered the possible tension between the value of $\sin 2\beta$ predicted from $|\epsilon_K|$ and $\Delta M_s/\Delta M_d$ in the SM and its direct measurement from $S_{J/\psi K_S}$, raised by the authors of refs. [21, 37–39]. We illustrated how CMM effects can remove this tension, and simultaneously reduce the $2.2\sigma$ discrepancy observed recently in the $B_s - \overline{B}_s$ mixing phase.

## Acknowledgments

We are grateful to Ulrich Nierste for initiating the project and to Waldemar Martens for his help with the *Mathematica* code of ref. [12]. We also thank Christopher Smith and Scott Willenbrock for useful discussions. This work is supported by the DFG grant No. NI 1105/1-1, by the DFG-SFB/TR9, by the Graduiertenkolleg "Hochenergiephysik und Astroteilchenphysik", and by the EU contract No. MRTN-CT-2006-035482 (FlaviaNet).

## A Loop functions

$$S_0(x_c) = x_c, \tag{A.1}$$

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \log(x_t)}{2(1-x_t)^3}, \tag{A.2}$$

$$S_0(x_c, x_t) = x_c \left[\log\frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \log x_t}{4(1-x_t)^2}\right], \tag{A.3}$$

$$F(x,y) = -\frac{1}{(x-1)(y-1)} - \frac{1}{x-y}\left[\frac{x\ln x}{(x-1)^2} - \frac{y\ln y}{(y-1)^2}\right], \tag{A.4}$$

$$G(x,y) = \frac{1}{(x-1)(y-1)} + \frac{1}{x-y}\left[\frac{x^2\ln x}{(x-1)^2} - \frac{y^2\ln y}{(y-1)^2}\right], \tag{A.5}$$



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$$L_0(x,y) = \frac{11}{18}G(x,y) - \frac{2}{9}F(x,y), \tag{A.6}$$

$$S^{(\tilde{g})}(x,y) = L_0(x,x) - 2L_0(x,y) + L_0(y,y). \tag{A.7}$$

## References


[1] J.C. Pati and A. Salam, *Unified lepton-hadron symmetry and a gauge theory of the basic interactions*, *Phys. Rev.* **D 8** (1973) 1240 [SPIRES].

[2] H. Georgi and S.L. Glashow, *Unity of all elementary particle forces*, *Phys. Rev. Lett.* **32** (1974) 438 [SPIRES].

[3] H. Georgi, *The state of the art — gauge theories*, talk in *Particles and fields*, C. Carlson ed., *AIP Conf. Proc.* **23** (1975) 575 [SPIRES];
H. Fritzsch and P. Minkowski, *Unified interactions of leptons and hadrons*, *Annals Phys.* **93** (1975) 193 [SPIRES].

[4] H. Georgi and S.L. Glashow, *Gauge theories without anomalies*, *Phys. Rev.* **D 6** (1972) 429 [SPIRES].

[5] P. Minkowski, $\mu \to e\gamma$ *at a rate of one out of* 1*-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421 [SPIRES].

[6] L.J. Hall and L. Randall, *Weak scale effective supersymmetry*, *Phys. Rev. Lett.* **65** (1990) 2939 [SPIRES];
G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavour violation: an effective field theory approach*, *Nucl. Phys.* **B 645** (2002) 155 [hep-ph/0207036] [SPIRES];
G. Colangelo, E. Nikolidakis and C. Smith, *Supersymmetric models with minimal flavour violation and their running*, *Eur. Phys. J.* **C 59** (2009) 75 [arXiv:0807.0801] [SPIRES];
B. Grinstein, V. Cirigliano, G. Isidori and M.B. Wise, *Grand unification and the principle of minimal flavor violation*, *Nucl. Phys.* **B 763** (2007) 35 [SPIRES].

[7] A.J. Buras, P. Gambino, M. Gorbahn, S. Jäger and L. Silvestrini, $\epsilon'/\epsilon$ *and rare* $K$ *and* $B$ *decays in the MSSM*, *Nucl. Phys.* **B 592** (2001) 55 [hep-ph/0007313] [SPIRES].

[8] A.H. Chamseddine, R.L. Arnowitt and P. Nath, *Locally supersymmetric grand unification*, *Phys. Rev. Lett.* **49** (1982) 970 [SPIRES];
R. Barbieri, S. Ferrara and C.A. Savoy, *Gauge models with spontaneously broken local supersymmetry*, *Phys. Lett.* **B 119** (1982) 343 [SPIRES].

[9] D. Chang, A. Masiero and H. Murayama, *Neutrino mixing and large CP-violation in B physics*, *Phys. Rev.* **D 67** (2003) 075013 [hep-ph/0205111] [SPIRES].

[10] See e.g. S. Baek, T. Goto, Y. Okada and K.-I. Okumura, *Neutrino oscillation, SUSY GUT and B decay*, *Phys. Rev.* **D 63** (2001) 051701 [hep-ph/0002141] [SPIRES];
T. Moroi, *CP violation in* $B_d \to \Phi K_S$ *in SUSY GUT with right-handed neutrinos*, *Phys. Lett.* **B 493** (2000) 366 [hep-ph/0007328] [SPIRES];
R. Harnik, D.T. Larson, H. Murayama and A. Pierce, *Atmospheric neutrinos can make beauty strange*, *Phys. Rev.* **D 69** (2004) 094024 [hep-ph/0212180] [SPIRES];
J. Hisano and Y. Shimizu, *GUT relation in neutrino induced flavor physics in SUSY* SU(5) *GUT*, *Phys. Lett.* **B 565** (2003) 183 [hep-ph/0303071] [SPIRES];
M. Ciuchini, A. Masiero, L. Silvestrini, S.K. Vempati and O. Vives, *Grand unification of quark and lepton FCNCs*, *Phys. Rev. Lett.* **92** (2004) 071801 [hep-ph/0307191] [SPIRES];
K. Cheung, S.K. Kang, C.S. Kim and J. Lee, *Correlation between lepton flavor violation and* $B_{(d,s)}$*-*$\bar{B}_{(d,s)}$ *mixing in SUSY GUT*, *Phys. Lett.* **B 652** (2007) 319 [hep-ph/0702050] [SPIRES].




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[11] S. Jäger and U. Nierste, *$B_s$-$\bar{B}_s$ mixing in an* SO(10) *SUSY GUT model*, *Eur. Phys. J.* **C 33** (2004) S256 [SPIRES].

[12] S. Jäger, M. Knopf, W. Martens, U. Nierste, C. Scherrer and S. Wiesenfeldt, *Flavor physics in an* SO(10) *grand unified model*, in preparation.

[13] T. Moroi, *Effects of the right-handed neutrinos on $\Delta S = 2$ and $\Delta B = 2$ processes in supersymmetric* SU(5) *model*, *JHEP* **03** (2000) 019 [hep-ph/0002208] [SPIRES];
N. Akama, Y. Kiyo, S. Komine and T. Moroi, *CP violation in kaon system in supersymmetric* SU(5) *model with seesaw-induced neutrino masses*, *Phys. Rev.* **D 64** (2001) 095012 [hep-ph/0104263] [SPIRES].

[14] R. Barbieri, L.J. Hall and A. Strumia, *Hadronic flavor and CP-violating signals of superunification*, *Nucl. Phys.* **B 449** (1995) 437 [hep-ph/9504373] [SPIRES].

[15] J.R. Ellis and M.K. Gaillard, *Fermion masses and Higgs representations in* SU(5), *Phys. Lett.* **B 88** (1979) 315 [SPIRES].

[16] S. Baek, T. Goto, Y. Okada and K.-I. Okumura, *Muon anomalous magnetic moment, lepton flavor violation and flavor changing neutral current processes in SUSY GUT with right-handed neutrino*, *Phys. Rev.* **D 64** (2001) 095001 [hep-ph/0104146] [SPIRES];
P. Ko, J.-H. Park and M. Yamaguchi, *Sflavor mixing map viewed from a high scale in supersymmetric* SU(5), *JHEP* **11** (2008) 051 [arXiv:0809.2784] [SPIRES];
F. Borzumati and T. Yamashita, *Minimal supersymmetric* SU(5) *model with nonrenormalizable operators: seesaw mechanism and violation of flavour and CP*, arXiv:0903.2793 [SPIRES].

[17] B. Bajc, P. Fileviez Perez and G. Senjanović, *Proton decay in minimal supersymmetric* SU(5), *Phys. Rev.* **D 66** (2002) 075005 [hep-ph/0204311] [SPIRES];
D. Emmanuel-Costa and S. Wiesenfeldt, *Proton decay in a consistent supersymmetric* SU(5) *GUT model*, *Nucl. Phys.* **B 661** (2003) 62 [hep-ph/0302272] [SPIRES].

[18] S. Wiesenfeldt, *Operator analysis for proton decay in SUSY* SO(10) *GUT models*, *Phys. Rev.* **D 71** (2005) 075006 [hep-ph/0501223] [SPIRES];
S.M. Barr and I. Dorsner, *A prediction from the type-III see-saw mechanism*, *Phys. Lett.* **B 632** (2006) 527 [hep-ph/0507067] [SPIRES].

[19] J.A. Casas, A. Ibarra and F. Jimenez-Alburquerque, *Hints on the high-energy seesaw mechanism from the low-energy neutrino spectrum*, *JHEP* **04** (2007) 064 [hep-ph/0612289] [SPIRES];
J. Sayre and S. Wiesenfeldt, *Naturalness and the neutrino matrix*, *Phys. Rev.* **D 77** (2008) 053005 [arXiv:0711.1687] [SPIRES].

[20] A.J. Buras, *Flavour physics and CP-violation*, hep-ph/0505175 [SPIRES];
K. Anikeev et al., *B physics at the Tevatron: run II and beyond*, hep-ph/0201071 [SPIRES].

[21] A.J. Buras and D. Guadagnoli, *Correlations among new CP-violating effects in $\Delta F = 2$ observables*, *Phys. Rev.* **D 78** (2008) 033005 [arXiv:0805.3887] [SPIRES].

[22] A. Lenz and U. Nierste, *Theoretical update of $B_s$-$\bar{B}_s$ mixing*, *JHEP* **06** (2007) 072 [hep-ph/0612167] [SPIRES].

[23] Particle Data Group collaboration, C. Amsler et al., *Review of particle physics*, *Phys. Lett.* **B 667** (2008) 1 [SPIRES];
Updates on *Particle Data Group homepage*, http://pdg.lbl.gov.

[24] Heavy Flavor Averaging Group collaboration, E. Barberio et al., *Averages of b−hadron and c−hadron Properties at the End of 2007*, arXiv:0808.1297 [SPIRES];

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Updates on *Heavy Flavor Averaging Group homepage*, http://www.slac.stanford.edu/xorg/hfag.

[25] S. Herrlich and U. Nierste, *The complete $|\Delta S| = 2$ Hamiltonian in the next-to-leading order*, *Nucl. Phys.* **B 476** (1996) 27 [hep-ph/9604330] [SPIRES].

[26] A.J. Buras, M. Jamin and P.H. Weisz, *Leading and next-to-leading QCD corrections to $\epsilon$ parameter and $B^0$-$\bar{B}^0$ mixing in the presence of a heavy top quark*, *Nucl. Phys.* **B 347** (1990) 491 [SPIRES].

[27] FlaviaNet Working Group on Kaon Decays collaboration, M. Antonelli et al., *Precision tests of the Standard Model with leptonic and semileptonic kaon decays*, arXiv:0801.1817 [SPIRES];
*Working Group on Precise SM Tests in K Decays homepage*, http://www.lnf.infn.it/wg/vus/.

[28] V. Lubicz and C. Tarantino, *Flavour physics and lattice QCD: averages of lattice inputs for the Unitarity Triangle Analysis*, *Nuovo Cim.* **123B** (2008) 674 [arXiv:0807.4605] [SPIRES].

[29] M. Battaglia et al., *The CKM matrix and the unitarity triangle*, in *Proceedings of the Workshop on CKM Unitarity Triangle*, CERN, Geneva Switzerland February 13–16 2002, [hep-ph/0304132] [SPIRES].

[30] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [hep-ph/9512380] [SPIRES].

[31] K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, *RunDec: a Mathematica package for running and decoupling of the strong coupling and quark masses*, *Comput. Phys. Commun.* **133** (2000) 43 [hep-ph/0004189] [SPIRES].

[32] HPQCD collaboration, I. Allison et al., *High-precision charm-quark mass from current-current correlators in lattice and continuum QCD*, *Phys. Rev.* **D 78** (2008) 054513 [arXiv:0805.2999] [SPIRES].

[33] Tevatron Electroweak Working Group collaboration, *Combination of CDF and D0 results on the mass of the top quark*, arXiv:0808.1089 [SPIRES].

[34] CKMfitter Group collaboration, J. Charles et al., *CP violation and the CKM matrix: assessing the impact of the asymmetric B factories*, *Eur. Phys. J.* **C 41** (2005) 1 [hep-ph/0406184] [SPIRES];
Updates on *CKM fitter package homepage*, http://ckmfitter.in2p3.fr.

[35] M. Drees, *Intermediate scale symmetry breaking and the spectrum of super partners in superstring inspired supergravity models*, *Phys. Lett.* **B 181** (1986) 279 [SPIRES].

[36] LEP Working Group for Higgs boson searches collaboration, R. Barate et al., *Search for the standard model Higgs boson at LEP*, *Phys. Lett.* **B 565** (2003) 61 [hep-ex/0306033] [SPIRES].

[37] E. Lunghi and A. Soni, *Possible indications of new physics in $B_d$-mixing and in $\sin(2\beta)$ determinations*, *Phys. Lett.* **B 666** (2008) 162 [arXiv:0803.4340] [SPIRES].

[38] A.J. Buras and D. Guadagnoli, *On the consistency between the observed amount of CP-violation in the $K-$ and $B_d$-systems within minimal flavor violation*, arXiv:0901.2056 [SPIRES].

[39] E. Lunghi and A. Soni, *Hints for the scale of new CP-violating physics from B-CP anomalies*, arXiv:0903.5059 [SPIRES].

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